4.3 The Ar ion laser The argon-ion laser can provide powerful CW visible coherent radiation of several watts. The laser operation is achieved as follows: The Ar atoms are ionized by electron collisions in a high current electrical discharge. Further multiple collisions with electrons excite the argon ion, Ar^+, to a group of 4*p* energy levels ~35 eV above the atomic ground state as shown in Figure 4Q3. Thus a population inversion forms between the 4*p* levels and the 4*s* level which is about 33.5 eV above the Ar atom ground level. Consequently, the stimulated radiation from the 4*p* levels down to the 4*s* level contains a series of wavelengths ranging from 351.1 nm to 528.7 nm. Most of the power however is concentrated, approximately equally, in the 488 and 514.5 nm emissions. The Ar^+ ion at the lower laser level (4*s*) returns to its neutral atomic ground state via a radiative decay to the Ar^+ ion ground state, followed by recombination with an electron to form the neutral atom. The Ar atom is then ready for "pumping" again.

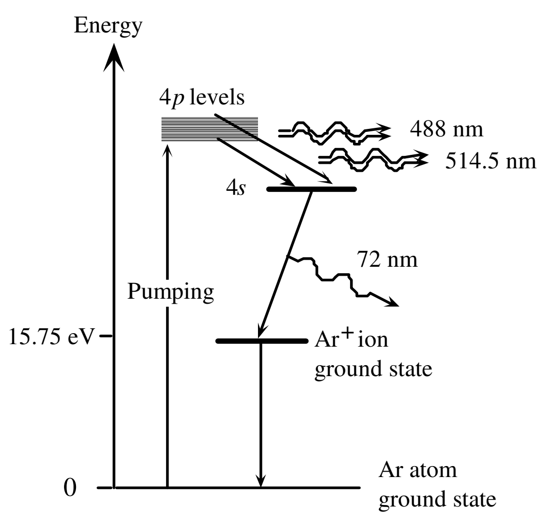
a Calculate the energy drop involved in the excited Ar^+ ion when it is stimulated to emit the radiation at 514.5 nm.

b The Doppler broadened linewidth of the 514.5 nm radiation is about 3500 MHz ($\Delta\upsilon$) and is between the half-intensity points.

(i) Calculate the Doppler broadened width in the wavelength; $\Delta\lambda$

(ii) Estimate the operation temperature of the argon ion gas; give the temperature in °C

c In a particular argon-ion laser the discharge tube, made of Beryllia (Beryllium Oxide), is 30 cm long and has a bore of 3 mm in diameter. When the laser is operated with a current of 40 A at 200 V dc, the total output power in the emitted radiation is 3 W. What is the efficiency of the laser?



The Ar-ion laser energy diagram

Figure 4Q3

a Given wavelength λ_o = 514.5 nm or 514.5 × 10^{-9} m, the change in the energy is

$$\Delta E = hc/\lambda_o = (6.626 \times 10^{-34} \text{ J s})(3.0 \times 10^{8} \text{ m s}^{-1})/(514.5 \times 10^{-9} \text{ m})$$

i.e. ΔE = **3.86 × 10^{-19} J** or **2.41 eV**

b The center frequency of the lasing emission is

$$\upsilon_o = \frac{c}{\lambda_o} = \frac{(3.0\times10^{8}\ \text{m s}^{-1})}{(514.5\times10^{-9}\ \text{m})} = 5.831 \times 10^{14}\ \text{s}^{-1}$$

The relationship between wavelength and frequency linewidths is

$$\Delta\lambda = \frac{\lambda_o}{\upsilon_o}\Delta\upsilon = \frac{(514.5\times10^{-9}\ \text{m})}{(5.831\times10^{14}\ \text{s}^{-1})}(3500\times10^{6}\ \text{s}^{-1})$$

∴ $\Delta\lambda$ = **3.09 × 10^{-12} m** or **0.00309 nm**

The frequency linewidth $\Delta\upsilon = 3500 \times 10^6\ \text{s}^{-1}$ is due to Doppler broadening. From $\Delta\upsilon$ we can find the operating temperature by using,

$$\Delta\upsilon = 2\upsilon_o\sqrt{\frac{2k_BT\ln(2)}{Mc^2}}$$

in which M is the mass of the Ar ion, that is, M = 39.95 g mol^{-1} or 6.636×10^{-26} kg.

$$\therefore \qquad T = \frac{Mc^2}{8k_B\ln(2)}\left(\frac{\Delta\upsilon}{\upsilon_o}\right)^2 = \mathbf{2810\ K = 2537\ °C}$$

which is the gas temperature. The tube gets very hot. Indeed, beryllia (BeO) is used because the operating temperature of the argon ion laser is high and beryllia has a high melting temperature and good thermal conductivity.

c The output power from the laser is 3 W. The input power is IV or (40 A)(200 V).

$$\textbf{Efficiency (\%)} = \frac{\text{Output Power from Laser}}{\text{Input Power into Laser}}\times100\% = \frac{3\ \text{W}}{(40\ \text{A})(200\ \text{V})}\times100\% = \mathbf{0.0375\%}$$

4.4 Einstein coefficients and critical photon concentration $\rho(h\upsilon)$ is the energy of the electromagnetic radiation per unit volume per unit frequency due to photons with energy $h\upsilon = E_2 - E_1$. Suppose that there are n_{ph} photons per unit volume. Each has an energy $h\upsilon$. The frequency range of emission is $\Delta\upsilon$. Then,

$$\rho(h\upsilon) = \frac{n_{ph}h\upsilon}{\Delta\upsilon}$$

Consider the Ar ion laser system. Given that the emission wavelength is at 488 nm and the linewidth in the output spectrum is 5×10^9 Hz (between half intensity points, that is $\Delta\upsilon$ = 2×5×10^9 Hz), *estimate* the photon concentration necessary to achieve more stimulated emission than spontaneous emission.

Solution

Suppose that there are n_{ph} photons per unit volume. Each has an energy $h\upsilon$. The frequency range of emission is $\Delta\upsilon$ as is the linewidth between half intensity points. Then,

$$\rho(h\upsilon) = \frac{n_{ph} h\upsilon}{\Delta\upsilon}$$

Now, $$\frac{R_{21}(\text{stim})}{R_{21}(\text{spon})} = \frac{c^3}{8\pi h\upsilon^3}\rho(h\upsilon) > 1$$

$\therefore$ $$\rho(h\upsilon) > \frac{8\pi h\upsilon^3}{c^3} = \frac{8\pi h}{\lambda^3} = \frac{8\pi(6.62\times10^{-34}\ \text{J s})}{(488\times10^{-9}\ \text{m})^3} = 1.43\times10^{-13}\ \text{J s m}^{-3}$$

$\therefore$ $$n_{ph} = \frac{\Delta\upsilon\rho(h\upsilon)}{h\upsilon} = \frac{(2\times5\times10^{9}\ \text{s}^{-1})(1.43\times10^{-13}\ \text{J s m}^{-3})}{(6.62\times10^{-34}\ \text{J s})(\dfrac{3\times10^{8}\ \text{m s}^{-1}}{433\times10^{-9}\ \text{m}})}$$

$\therefore$ n_{ph} = **3.5×10^{15} Photons m^{-3}**.

Note that this represents the critical photon concentration for stimulated emission to just exceed spontaneous emission in the absence of any photon losses. It does *not* represent the photon concentration for laser operation. In practice, the photon concentration is much greater during laser operation.

4.6 Threshold gain and population inversion

a Consider a He-Ne gas laser operating at 632.8 nm. The tube length L = 40 cm, tube diameter is 1.5 mm and mirror reflectances are approximately 99.9% and 98%. The linewidth $\Delta\upsilon$ = 1.5 GHz, the loss coefficient is $\gamma \approx 0.05$ m^{-1}, spontaneous decay time constant $\tau_{sp} = 1/A_{21} \approx$ 300 ns, $n \approx 1$. What is the threshold gain and population inversion?

b Consider a 488 nm Ar-ion gas laser. The tube length L = 1 m, tube mirror reflectances are approximately 99.9% and 95%. The linewidth $\Delta\upsilon$ = 3 GHz, the loss coefficient is $\gamma \approx 0.1$ m^{-1}, spontaneous decay time constant $\tau_{sp} = 1/A_{21} \approx$ 10 ns, $n \approx 1$. What is the threshold population inversion?

c Consider a semiconductor laser operating at (λ_o) 870 nm with a GaAs laser cavity with cleaved facets. The cavity length is 50 μm. The refractive index (n) of GaAs is 3.6. The loss coefficient γ at normal temperatures is of the order of ~10 cm^{-1} Calculate the required threshold gain. What is your conclusion?

[Note: γ depends on a number of factors including the injected carrier concentration and at best the above calculation is an estimate. We cannot simply calculate the threshold population inversion, $\Delta N_{th} = \Delta n_{th}$, from Eq. (9) in Section 4.9 which does not apply for a number of reasons; see Section 7.2 in P. Bhattacharya, *Semiconductor Optoelectronic Devices, Second Edition* (Prentice-Hall, New York, 1993).

Solution

a $$g_{th} = \gamma - \frac{1}{2L}\ln(R_1R_2) = 0.05\ \text{m}^{-1} - \frac{1}{2(0.4\ \text{m})}\ln(1\times0.99) = \mathbf{0.077\ m^{-1}}.$$

The emission frequency $\upsilon_o = c/\lambda_o$ = (3×10^{8} ms^{-1})/(632.8×10^{-9} m) = 4.74×10^{15} s^{-1}. From laser characteristics, and

$$\Delta N_{th} \approx g_{th} \frac{8\pi \upsilon_o^2 \eta^2 \tau_{sp} \Delta \upsilon}{c^2}$$

$$= (0.077 \text{ m}^{-1}) \frac{8\pi(4.74\times10^{14} \text{ s}^{-1})^2 (1)^2 (300\times10^{-9} \text{ s})(1.5\times10^9 \text{ s}^{-1})}{(3\times10^8 \text{ m s}^{-1})^2}$$

$$= \mathbf{2.1\times10^{15}\ m^{-3}}.$$

Note: The number of Ne atoms per unit volume n_{Ne} can be found from the Gas law using the partial pressure of Ne:

$$P_{\text{Ne}} V = \frac{N_{\text{Ne}}}{N_A} RT$$

∴

$$n_{\text{Ne}} = \frac{N_{\text{Ne}}}{V} = \frac{N_A P_{\text{He}}}{RT} \approx \frac{(6.02\times10^{23} \text{ mol}^{-1})(\frac{1}{6} 2.5 \text{ torr}\times10^5 \frac{\text{N m}^{-2}}{\text{torr}})}{(8.315\times10^{23} \text{ J K}^{-1}\text{mol}^{-1})(300 \text{ K})}$$

$$\approx \mathbf{1.3\times10^{22}\ m^{-3}}.$$

b

$$g_{th} = \gamma - \frac{1}{2L}\ln(R_1R_2) = 0.1 \text{ m}^{-1} - \frac{1}{2(1 \text{ m})}\ln(0.999\times0.95) = 0.126 \text{ m}^{-1}.$$

The emission frequency $\upsilon_o = c/\lambda_o = (3\times10^8 \text{ ms}^{-1})/(488\times10^{-9} \text{ m}) = 6.14\times10^{15} \text{ s}^{-1}$. From laser characteristics,

and

$$\Delta N_{th} \approx g_{th} \frac{8\pi \upsilon_o^2 n^2 \tau_{sp} \Delta \upsilon}{c^2}$$

$$= (0.126 \text{ m}^{-1}) \frac{8\pi(6.14\times10^{14} \text{ s}^{-1})^2 (1)^2 (10\times10^{-9} \text{ s})(3\times10^9 \text{ s}^{-1})}{(3\times10^8 \text{ m s}^{-1})^2}$$

$$= \mathbf{4.0\times10^{14}\ m^{-3}}.$$

c

$$g_{th} = \gamma - \frac{1}{2L}\ln(R_1R_2) = 1000 \text{ m}^{-1} - \frac{1}{2(50\times10^{-6} \text{ m})}\ln(0.32\times0.32)$$

$$= \mathbf{2.4\times10^{4}\ m^{-1}}.$$

Substantially larger than a gas laser!

4.11 InGaAsP-InP Laser Consider a InGaAsP-InP laser diode which has an optical cavity of length 250 microns. The peak radiation is at 1550 nm and the refractive index of InGaAsP is 4. The optical gain bandwidth (as measured between half intensity points) will normally depend on the pumping current (diode current) but for this problem assume that it is 2 nm.

a What is the mode integer m of the peak radiation?

b What is the separation between the modes of the cavity?

c How many modes are there in the cavity?

d What is the reflection coefficient and reflectance at the ends of the optical cavity (faces of the InGaAsP crystal)?

e What determines the angular divergence of the laser beam emerging from the optical cavity?

Solution

a The wavelength λ of a cavity mode and length L are related by

$$m\frac{\lambda}{2n} = L$$

so that $$m = \frac{2nL}{\lambda} = \frac{2(4)(250\times10^{-6})}{(1550\times10^{-9})} = 1290.3.$$

When m = 1290, $\lambda = 2nL/m$ = 1550.39 nm so that the peak radiation has m = 1290.

b Mode separation is given by,

$$\Delta\lambda_m = \frac{\lambda^2}{2nL} = \frac{(1550\times10^{-9})^2}{2(4)(250\times10^{-6})} = 1.20 \text{ nm}$$

c <u>The linewidth is 2 nm</u>

Let the optical linewidth $\Delta\lambda$ be between λ_1 and λ_2. Then $\lambda_1 = \lambda - (^1/_2)\Delta\lambda$ = 1549 nm and $\lambda_2 = \lambda + (^1/_2)\Delta\lambda$ = 1551 nm and the mode numbers corresponding to these are

$$m_1 = \frac{2nL}{\lambda_1} = \frac{2(4)(250\times10^{-6})}{(1549\times10^{-9})} = 1291.15$$

$$m_2 = \frac{2nL}{\lambda_2} = \frac{2(4)(250\times10^{-6})}{(1551\times10^{-9})} = 1289.49$$

Now m must be an integer and the corresponding wavelength must fit into the optical gain curve.

Taking m = 1290, gives $\lambda = 2nL/m$ = 1550.39 nm; within optical gain 1549 – 1551 nm

Taking m = 1291, gives $\lambda = 2nL/m$ = 1549.18 nm; within optical gain 1549 – 1551 nm

Taking m = 1289, gives $\lambda = 2nL/m$ = 1551.59 nm; just outside optical gain 1549 – 1551 nm

There are **2 modes.**

d $r = (n - 1)/(n + 1) = (4 - 1)/(4 + 1) = 0.6$

$R = r^2 = 0.36$ or 36%.

e Diffraction at the active region cavity end.